

A hyperbola has a focus at the pole and vertices with rectangular co-ordinates $(-3, 0)$ and $(-7, 0)$.

SCORE: ____ / 25 PTS

- [a] Find polar co-ordinates for the vertices, using positive values of r and θ . NOTE: You do NOT need to show work.

$$\textcircled{2} \quad (3, \pi) \quad (7, \pi) \quad \textcircled{2}$$

- [b] Find the polar equation of the hyperbola.

$$\textcircled{6} \quad r = \frac{ep}{1 - e \cos \theta}$$

$$r = \left| \frac{\frac{5}{2}, \frac{21}{5}}{1 - \frac{5}{2} \cos \theta} \right| \cdot \frac{2}{2}$$

$$\textcircled{2} \quad r = \frac{21}{2 - 5 \cos \theta}$$

$$e = \frac{PF}{PQ} = \frac{P'F}{P'Q}$$

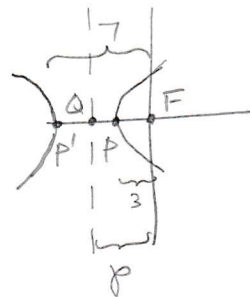
$$e = \left| \frac{3}{p-3} = \frac{7}{7-p} \right| \quad \textcircled{5}$$

$$21 - 3p = 7p - 21$$

$$42 = 10p$$

$$p = \frac{21}{5} \quad \textcircled{3}$$

$$e = \frac{3}{\frac{21}{5} - 3} \cdot \frac{5}{5} = \frac{15}{21 - 15} = \frac{15}{6} = \frac{5}{2} \quad \textcircled{2}$$



Find the logarithmic formula for $\sinh^{-1} x$ by solving $x = \sinh y$ for y

SCORE: ____ / 25 PTS

using the exponential definition and an algebraic substitution $z = e^y$ (or a similar substitution).

$$\textcircled{5} \quad x = \frac{e^y - e^{-y}}{2} = \frac{z - \frac{1}{z}}{2} \cdot \frac{z}{z} = \frac{z^2 - 1}{2z} \quad \textcircled{3}$$

$$2xz = z^2 - 1$$

$$\textcircled{5} \quad 0 = z^2 - 2xz - 1$$

$$\textcircled{6} \quad z = \frac{2x \pm \sqrt{4x^2 + 4}}{2} = \frac{2x \pm 2\sqrt{x^2 + 1}}{2} = x \pm \sqrt{x^2 + 1} \quad \textcircled{3}$$

$$e^y = x + \sqrt{x^2 + 1}$$

$$y = \ln(x + \sqrt{x^2 + 1}) = \sinh^{-1} x \quad \textcircled{3}$$

Name the shape of the graphs of the following polar equations. Be as specific as possible.

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If the graph is a rose curve, state the number of petals.

[a] $r = 9 + 5\sin\theta$ ④ LIMACON WITH DIMPLE [b]

$r = 5\sin 4\theta$ ④ ROSE CURVE (8 PETALS)

[c] $r = 5\cos\theta$ ③ CIRCLE [d]

$r = \frac{9}{4 + 5\sin\theta}$ ③ HYPERBOLA

[e] $r = \frac{9}{5 - 4\cos\theta}$ ③ ELLIPSE [f]

$r = 5 - 5\cos\theta$ ③ CARDIOID

AJ throws a football from an initial height of 4 feet, at 18 feet per second, at an angle of 60° with the horizontal. SCORE: ____ / 15 PTS
Write parametric equations for the position of the football.

$$x = (v_0 \cos \theta) t$$

$$y = h + (v_0 \sin \theta) t - 16t^2$$

$$v_0 = 18$$

$$\theta = 60^\circ$$

$$h = 4$$

$$\textcircled{5} \quad x = (18 \cos 60^\circ) t$$

$$\textcircled{5} \quad y = 4 + (18 \sin 60^\circ) t - 16t^2$$

→

$$x = 9t \quad \textcircled{2\frac{1}{2}}$$

$$y = 4 + 9\sqrt{3}t - 16t^2 \quad \textcircled{2\frac{1}{2}}$$

Rewrite $\operatorname{csch}\left(-\frac{1}{2}\ln x\right)$ in terms of exponential functions and simplify.

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$$\left[\frac{2}{e^{-\frac{1}{2}\ln x} - e^{\frac{1}{2}\ln x}} \right] \textcircled{3}$$

$$= \left[\frac{2}{\frac{1}{\sqrt{x}} - \sqrt{x}} \right] \textcircled{4} \cdot \frac{\sqrt{x}}{\sqrt{x}}$$

$$= \left[\frac{2\sqrt{x}}{1-x} \right] \textcircled{3}$$

BJ and CJ were working on their polar graphing partner quiz.

SCORE: ____ / 40 PTS

On the question about the polar equation $r = 2\sqrt{3} - 4\sin 3\theta$, they determined correctly that the symmetry tests $(-r, \theta)$, $(r, -\theta)$, $(-r, \pi - \theta)$ and $(-r, -\theta)$ do NOT indicate that the graph is symmetric.

POLE AXIS AXIS $\theta = \frac{\pi}{2}$

- [a] Using their results, along with the tests and shortcuts shown in lecture, test if the graph is symmetric over the pole, the polar axis and/or $\theta = \frac{\pi}{2}$. State your conclusions in the table. NOTE: Run as FEW tests as needed to prove your answers are correct.

POLE: $(r, \pi + \theta)$

$$r = 2\sqrt{3} - 4\sin 3(\pi + \theta) \quad (4)$$

$$r = 2\sqrt{3} - 4\sin(3\pi + 3\theta)$$

$$r = 2\sqrt{3} - 4(\sin 3\pi \cos 3\theta + \cos 3\pi \sin 3\theta)$$

$$r = 2\sqrt{3} + 4\sin 3\theta \quad (4)$$

$\theta = \frac{\pi}{2}: (r, \pi - \theta)$

$$r = 2\sqrt{3} - 4\sin 3(\pi - \theta) \quad (4)$$

$$r = 2\sqrt{3} - 4(\sin 3\pi \cos 3\theta - \cos 3\pi \sin 3\theta)$$

$$r = 2\sqrt{3} - 4\sin 3\theta \quad (4)$$

Type of symmetry	Conclusion
Over the pole	CAN'T TELL
Over the polar axis	CAN'T TELL
Over $\theta = \frac{\pi}{2}$	SYM

(4)

- [b] Based on the results of part [a], what is the minimum interval of the graph you need to plot (before using reflections to draw the rest of the graph)?

$$\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \quad (5)$$

- [c] Find all angles algebraically in the minimum interval in part [b] at which the graph goes through the pole.

$$(5) \quad 2\sqrt{3} - 4\sin 3\theta = 0$$

$$\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$(2\frac{1}{2}) \quad \sin 3\theta = \frac{\sqrt{3}}{2}$$

$$3\theta \in \left[-\frac{3\pi}{2}, \frac{3\pi}{2}\right]$$

$$(5) \quad 3\theta = -\frac{4\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$$

$$(2\frac{1}{2}) \quad \theta = -\frac{4\pi}{9}, \frac{\pi}{9}, \frac{2\pi}{9}$$

Eliminate the parameter for the parametric equations

$$x = 2 \cos t$$

$$y = \cos 2t$$

SCORE: ____ / 15 PTS

$$\textcircled{3} \quad \underline{\cos t = \frac{1}{2}x}$$

$$\textcircled{6} \quad \underline{y = 2 \cos^2 t - 1}$$

$$\textcircled{4} \quad \underline{y = 2\left(\frac{1}{2}x\right)^2 - 1}$$

$$\textcircled{2} \quad \underline{y = \frac{1}{2}x^2 - 1}$$